Some Remarks on the V-Particles*

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It is qualitatively investigated whether the abundance of V-particle production can be reconciled with their long lifetime by using only interactions of a conventional structure. This is possible, provided a V-particle is produced together with another heavy unstable particle (Sec. II). Two distinct groups of interactions are needed: for one, the coupling is strong (II); for the other, it is very weak (III). Two kinds of V-particles are considered, Fermions of mass $\sim 2200m$ and Bosons ($\sim 800m$). The arguments are somewhat different, according to whether the latter are nonpseudoscalar (III) or pseudoscalar (V). The competition with processes involving μ-mesons is discussed (IV). Possible connections with the τ-meson are commented on in Sec. V. The preliminary nature of the present analysis is stressed (VI).

I. INTRODUCTION

IN the last few years a sizable number of events have been recorded which have been given the collective name of V-particle decay.[1-10] While a number of characteristics of these phenomena do require more experimental elucidation, a few qualitative features are already standing out clearly. Among these we mention first of all:

(a) In high energy events V-particles are produced with a probability $\gtrsim 1$ percent of the π-meson production. Thus, the production is copious.

(b) These new particles have lifetimes $\gtrsim 10^{-10}$ sec.

(c) There is a marked dissymmetry between the neutral and the charged V's, the former ones being observed much more frequently. It is not quite clear yet whether this is due to the charged V's being produced in lesser abundance or whether they have a shorter lifetime. Some observations tend to support the latter view.[2]

One of the most striking aspects is certainly the long lifetime of the V-particles. In fact, if one would consider the same mechanism which produces them to be instrumental for their decay, one would estimate lifetimes τ of the order of 10^{-21} sec. Note in this connection also that the Q-values involved ($\sim 30-100$ Mev) are so large that a threshold argument cannot be invoked to explain the large τ.

In regard to what the nature of the decay products is, the situation is not quite clear. With certainty one has been able to identify protons as well as π-mesons among the decay products. In no single event that I know of has the identity of both observed V_0-decay products been established. Nor is it as yet a settled point whether we have to do with one (or more) three-body decays involving also neutral decay products, with a superposition of two (or more) two-body decays, or with a combination of processes of either type. Indeed, as Leighton has pointed out,[8] one must exercise caution in the use of the coplanarity argument in all cases where one of the decay products is a nucleon.

At the time of writing it is most commonly assumed that there exists at least one V_0 heavier and at least one lighter than the nucleon (masses $\sim 2200m$ and $800m$, respectively), both undergoing two-body decay.

It will be attempted in this paper to find a model for these phenomena which reconciles the copiousness of the production of the V's with their longevity. Only couplings of a conventional structure will be used, not

* The principal ideas of this paper were reported at the second Rochester Conference on Meson Physics (January 11 and 12, 1952).
[1] G. D. Rochester and C. C. Butler, Nature 160, 855 (1947).
[2] Seriff, Leighton, Hsiao, Cowan, and Anderson, Phys. Rev. 78, 290 (1950).
[3] G. McCusker and D. Millar, Nuovo cimento 8, 289 (1951).
[4] V. D. Hopper and S. Biswas, Phys. Rev. 80, 1099 (1950).
[5] R. Armenteros et al., Nature 167, 501 (1951).
[6] H. S. Bridge and M. Annis, Phys. Rev. 82, 445 (1951).
[7] Thompson, Cohn, and Flum, Phys. Rev. 83, 175 (1951).
[8] Leighton, Wanlass, and Alford, Phys. Rev. 83, 843 (1951).
[9] W. B. Fretter, Phys. Rev. 83, 1053 (1951).
[10] R. Armenteros et al., Phil. Mag. 42, 1113 (1951).

so much in the conviction that our present day type of field theory constitutes the adequate framework to represent mesonic and related phenomena, but rather in an endeavor to put these structures to the test. We shall start from the following assumptions:[11]

(1) The V_0 with mass $\sim$2200m is a Fermion. This is not certainly so, because (a) the decay product accompanying the proton has not definitely been identified as a π- or a μ-meson; (b) it is not certain whether or not, e.g., a neutrino is also emitted.

(2) The V_0 with mass $\sim$800m is a Boson. This is also tentative for very similar reasons. However, one picture has been obtained by Bridge[12] in Colorado in which both charged secondaries interact strongly with matter. Thus, it would seem implausible that one can simultaneously assume proton$+\mu$-meson for the 2200m particle and π-meson$+\mu$-meson for the 800m case.

(3) An assumption of symmetry concerning V_0 and V_{ch}: namely, that there are two V_{ch}, a Fermion and a Boson with respective masses not much different from the corresponding V_0's.

(4) In the absence of interactions, the Fermion wave functions shall satisfy a Dirac equation and the Boson wave functions a Schrödinger-Gordon-Klein equation.

Thus, the heavy Fermion is here as elementary as the nucleon. In this respect, the present model differs radically from any strong coupling isobar picture for the V-particles in which the 2200m particle appears as an essentially composite structure. So far it has not appeared possible to stabilize isobars sufficiently against transitions to the ground state, unless such selection rules are assumed to hold as seem rather artificial. This circumstance has, in fact, led to the exploration of the present alternative.

The following analysis rests further on the idea that the Fermion-Boson interactions between all particles mentioned above can be divided in two distinct groups.[11] The first group, discussed in the next section, comprises the nucleon-π-meson interaction and certain others of comparable strength. The second group (Sec. III) comprises very weak interactions between these same particles. The order of magnitude of the couplings is indeed reminiscent of those that were introduced by Yukawa to describe β-decay as being brought about through the intermediary of Bosons (see Sec. VI). As pointed out by Oppenheimer,[13] both groups can be characterized by saying that the rule of conservation of the number of nucleons (itself an *ad hoc* assumption needed for guaranteeing sufficient stability of matter) is generalized so as to refer to nucleons and the heavier V-particles collectively.

I will now consider first the production and thereafter

the decay mechanisms. The arguments will have to be qualitative in nature; the emphasis will be throughout on relative orders of magnitude and on the role of selection rules. Furthermore, certain generalizations of the Furry theorem which were discovered by the Japanese workers in their elaborate discussions of decay processes of heavy Bosons will often be used (see Sec. IIIAb). The present work also contains many elements that already appear in an extensive survey of V-particle models and that have recently been published in the *Progress of Theoretical Physics*;[14-17] especially, some of the interactions originally considered by Nambu and co-workers and by Ôneda also occur here.

II. THE PRODUCTION MECHANISMS OF THE V-PARTICLES

The schemes to be considered will all involve couplings of the "$\bar{\psi}\psi\phi$-type," i.e., bilinear in the Fermion wave function ψ and its adjoint $\bar{\psi}$ and linear in the Boson wave function ϕ. This itself is an assumption, and it cannot be asserted at this moment that we can derive very much confidence in this structure of the interaction from our experiences with the analogous π-meson-nucleon coupling. This is not the right place to discuss in any detail the problematics of the so-called unrenormalizable theories or of the complications arising from interactions involving large coupling constants. But it should be emphasized that we will operate here with these interactions in the same crude sense as is done in what we at present choose to call the theory of the π-meson-nucleon problems.

First a matter of notation: nucleons will be designated by N_0. In particular, the neutron will be denoted by $N_0{}^0$ and the proton by $N_0{}^+$. The Fermions of mass $\sim$2200 will be called N_1. $N_1{}^0$ is the V_0-Fermion; $N_1{}^\pm$ denotes the V_{ch}. The relation of $N_1{}^+$ to $N_1{}^-$ shall be that of particle to antiparticle. π-mesons (neutral, charged) shall be denoted by π_0, $(\pi_0{}^0, \pi_0{}^\pm)$, the Bosons of mass $\sim$800m by $\pi_1(\pi_1{}^0, \pi_0{}^\pm)$. This is a convenient shorthand for the following discussion.

One can now in principle admit Fermion-Boson couplings of the type mentioned above between all these various sets of particles. I denote them by $(N_iN_j\pi_k)$, $i, j, k=0, 1$. The interaction shall, of course, have the appropriate hermiticity and covariance properties, but the detailed structure of the coupling (pseudoscalar, scalar, etc.) is left unspecified for the moment with the exception of $(N_0N_0\pi_0)$. This is the nucleon-π-meson interaction, which shall of course be appropriate to the experimentally known fact that $\pi_0{}^0$ as well as $\pi_0{}^\pm$ are pseudoscalar particles.

It will now be assumed: $(N_iN_j\pi_k)$ can only be strong if

$$i+j+k=\text{even}. \tag{1}$$

[11] The question of the uniqueness of these assumptions is discussed in Sec. VI.

[12] Bridge, Courant, and Rossi, Phys. Rev. **85**, 159 (1952).

[13] J. R. Oppenheimer, discussion remark at the Rochester Conference.

[14] Y. Nambu *et al.*, Prog. Theor. Phys. **6**, 615, 619 (1951).

[15] K. Aizu and T. Kinoshita, Prog. Theor. Phys. **6**, 630 (1951).

[16] H. Miyazawa, Prog. Theor. Phys. **6**, 631 (1951).

[17] S. Ôneda, Prog. Theor. Phys. **6**, 633 (1951).

Condition (1) singles out the following couplings:[18]

$$(N_0 N_0 \pi_0), \qquad (2a)$$

$$(N_1 N_1 \pi_0), \qquad (2b)$$

$$(N_1 N_0 \pi_1). \qquad (2c)$$

However little we know about (2a), we do have a strongly coupled system here. More precisely, our assumption shall mean that (2b) and (2c) are comparable in strength with (2a). The coupling constants for all couplings (2) shall indiscriminately be denoted by the one symbol G. This does not mean a prejudice to the equality of all coupling strengths, on charge independence and the like. It shall merely indicate an order of magnitude. The scheme (2) clearly leads to matrix elements for the creation of N_1's and π_1's in high energy nucleon-nucleon collisions. According to (2) the V's are produced in pairs, pairs of either (N_1, N_1) or (N_i, π_1). In this connection we note:

(1) The couplings here considered are only between particles whose existence seems reasonably certain. Such an economy seems fair as a starting point. But it cannot at all be excluded that the $(N_i N_j \pi_k)$-scheme might prove tenable while yet the specific interactions (2) and (12) below would be of a too narrow scope. Thus the further discussion of the N_1-properties remains unaltered if the π_1 were a different Boson than the V_{800}, provided only that it is heavier than the $N_1 - N_0$ mass difference and has a long lifetime for decay into two or more π_0's. Conversely, retaining the present meaning of π_1, one may replace everywhere N_1 by a heavy Fermion N_1', which may be different from N_1 and must be different from N_0 without affecting the discussion of the π_1-properties. With the sole exception of the correlation between N_1- and π_1-production, mentioned in point 3 below, the entire contents of this paper can thus be taken over into a much wider framework of particles and interactions. It should be emphasized that the two points vital to the present discussion are (a) that the V's are produced together with another heavy unstable particle; it will therefore be decisive to know whether there exists production correlations between unstable particle pairs (which need not have lifetimes of the same order of magnitude), (b) that the couplings can be divided into strong ones of the type (2) and weak ones of the type (12) below.

(2) The possibility that the V's are produced in pairs is hard to rule out from the present experimental evidence.[19] Photographs showing more than one V have actually been obtained by McCusker and Millar,[3] by

Thompson,[20] by Rossi's group[20] and by Leighton.[20] Such pictures may, of course, not be considered as definite evidence for the present view, as plural rather than multiple production may occur. The most conclusive argument would naturally be the determination of the threshold for the production reaction.

(3) (2c) combined with (2a) leads to such reactions as

$$N_0{}^0 + N_0{}^0 \rightarrow N_1{}^0 + N_0{}^0 + \pi_1{}^0. \qquad (3a)$$

The lowest order matrix elements are $\sim G^3$. On the other hand, (2c) by itself gives, in order G^2,

$$N_0{}^0 + N_0{}^0 \rightarrow N_1{}^0 + N_1{}^0, \qquad (3b)$$

and this reaction has a lower threshold than (3a). Hence, there must be an energy region in which N_1-production is much favored over π_1-production. Assuming that for higher energies the cross sections for the processes (3a) and (3b) become comparable and that only one of the V's produced is actually picked up by the observation, the frequency ratio of $N_1{}^0$ to $\pi_1{}^0$ would be $\sim 3:1$. Leighton[8] quotes a ratio $\sim 2:1$ and Armenteros[10] $\sim 1.6 \pm 0.5$, which is somewhat lower[21] but seems to bear out a preponderance of $N_1{}^0$ over $\pi_1{}^0$.

In higher order, the coupling (2b) can combine with the other two to contribute also to the probabilities for the events (3a, b). This coupling is not strictly necessary for the N_1 or π_1 creation. It can, however, be classed among the strong interactions without upsetting the balance between copious production and long life-times.

(4) In the reactions (3a, b) both emerging Fermions will be particles (meant here as a contradistinction to antiparticles), if the colliding ones are particles. Hence the presumed particle nature of the initial N_0's leads to the singling out of the particle N_1's over the anti-particle N_1's. As the difference of the number of particles and antiparticles (whether N_0 or N_1), is a constant of the motion, one can only get an anti-N_1 if enough energy is available to create a pair of heavy Fermions. Hence the anti-N_1 should be at most equally frequent as the so far spectacularly absent antiproton.

(5) The production of charged N_1's in nucleon collisions depends decisively on whether $N_1{}^+$ is a particle and $N_1{}^-$ an antiparticle, or *vice versa*. This is a physically decidable distinction of the $N_1{}^\pm$-properties in relation to those of $N_0{}^\pm$. It is interesting to note that if the situation for the $N_1{}^{\mathrm{ch}}$ were the reverse of that for the $N_0{}^{\mathrm{ch}}$, i.e., if the relation of the $N_1{}^-$ to the $N_0{}^+$ is that of particle to particle, the only possible $2N_0 \rightarrow 2N_1$ reaction is the one given by (3b), always remembering the conservation law mentioned under (3). Neither a neutron-proton nor a proton-proton collision could then give rise to the production of two N_1's without violating

[18] The interaction (2b) has first been considered by Nambu (see reference 14). Interactions between the heavy V-particles, nucleons, and τ-mesons that correspond closely to interaction (2c) occur in some models of Nambu and in that of Ôneda (see reference 17). Also some of the interactions mentioned in (12) below occur in these papers.

[19] I am much indebted for discussions on this point with various experimentalists during the Rochester Conference.

[20] Reported at the Rochester Conference. Leighton's work was presented by W. B. Fretter.

[21] However, more recent observations reported at the Rochester Conference seem to indicate that there are relatively fewer π_1's than is stated in references 9 and 11.

either the conservation of charge or the conservation of (particles minus antiparticles). The production of N_1^- would, of course, be possible provided enough energy is available to produce mesons as well, which can carry off charge. Such processes have a threshold higher than the one for (3b), and hence there would be a dissymmetry in N_1^0 vs N_1^{ch} production favoring the former. Without going yet in any detail into the decay problems, it may already be remarked that in this situation the N_1^- can only decay into a neutron plus other particles, again in accordance with the generalized conservation law of the number of nucleons.

In any case, the counterpart of (3a) for the production of charged V's exists; we have

$$N_0^0 + N_0^0 \rightarrow N_1^- + N_0^0 + \pi_1^+, \tag{3c}$$

if N_1^- is a particle, and

$$N_0^0 + N_0^0 \rightarrow N_1^+ + N_0^0 + \pi_1^-, \tag{3d}$$

if N_1^+ is a particle.

The present argument is, perhaps, of somewhat more general interest, as it shows that negatively charged heavy Fermions may exist which can be produced more easily than antiprotons.
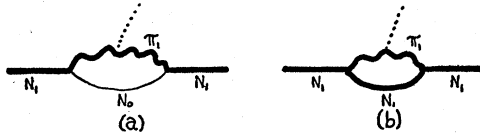

FIG. 1. Representative graphs leading to interactions of the type (5).

Too little is known to warrant at this moment a more detailed analysis of the production cross sections; such an investigation can for the rest be made along similar lines as is done for the π_0-production through (2a).[22]

It is clear that if (2) were the only couplings in the world then both the N_1 and the π_1 would be stable. Here the mass inequality

$$m(N_1) < m(N_0) + m(\pi_1)$$

is, of course, essential. But furthermore, we must immediately admit all electrically charged N_i and π_i to interact with the electromagnetic field. However, those interactions also in combination with (2) do not lead to any decay.

First of all, it is impossible that couplings of the type

$$i\epsilon \bar{\psi}_1 \gamma_\mu \psi_0 A_\mu + \text{h.c.} \tag{4}$$

exist. Here ϵ is the electric charge; ψ_1, ψ_0 are the wave functions of $N_1^\pm$, $N_0^\pm$, respectively; and A_μ is the electromagnetic potential. It is indeed readily seen that the presence of such "mixed" current densities occurring in (4) violates the law of conservation of charge. Thus,

[22] See K. M. Watson and K. A. Brueckner, Phys. Rev. 83, 1 (1951).

couplings of this type which would lead to $N_1 \rightarrow N_0 + \gamma$ need not concern us.

Of course, couplings with the electromagnetic field other than (4) can be envisaged, such as

$$\sim \bar{\psi}_1 \gamma_\mu \gamma_\nu \psi_1 F_{\mu\nu} + \text{h.c.} \tag{5}$$

$$\sim \bar{\psi}_1 \gamma_\mu \gamma_\nu \psi_0 F_{\mu\nu} + \text{h.c.} \tag{6}$$

Both (5) and (6) satisfy all covariance and gauge invariance requirements. Moreover, such interactions may be present for both $N_1^\pm$ and N_1^0, just as the effective anomalous magnetic moment interaction $\bar{\psi}_0 \gamma_\mu \gamma_\nu \psi_0 F_{\mu\nu}$ occurs for protons and neutrons. In fact, the existence of an interaction like (5) is already a consequence of the presence of either or both of (2b, c) and of the electromagnetic interaction of the charged particles of each kind which are involved. In Fig. 1 one of the typical graphs for each case is drawn. Here and in the following, these graphological conventions are used: a straight thin (heavy) line denotes an $N_0(N_1)$; a waved thin (heavy) line stands for a $\pi_0(\pi_1)$; and a dotted line for a photon.

However, interaction (5) cannot lead to radiative V-decay either. It is true that (6) could give rise to that phenomenon, but on the other hand (6) cannot be constructed from the interactions (2) combined with electromagnetic couplings. This is generally true to all orders in G and ϵ and is a consequence of the "conservation of evenness" of all the elementary couplings which we have introduced so far. Hence, if we take the general view that interactions like (5), (6) are only occurring insofar as they are derived from the elementary $\bar{\psi}\psi\phi$ couplings (and the $\phi^2 A_\mu$ couplings between Bosons and the electromagnetic field), it is a rational procedure to exclude the interaction (6), although it could always be introduced in an *ad hoc* fashion. (Actually we will introduce in Sec. III ($N_i N_j \pi_k$) couplings from which (6) can be derived in the above sense. But then the corresponding strength will be weak and the radiative decay mode will be insignificant compared to other decay schemes.)

It is also easy to see that the rule of evenness excludes the possibility of the transition

$$\pi_1^0 \rightarrow 2\gamma, \tag{7}$$

which would be the analog of the ordinary neutral π-meson decay. This is again true to all orders, and the same can be said for the following interactions (insofar as they are not already forbidden for other reasons):

$$\pi_1^0 \rightarrow \pi_0^0 + \gamma \tag{8}$$

$$\pi_1^0 \rightarrow \pi_0^+ + \pi_0^- \tag{9a}$$

$$\pi_1^0 \rightarrow \pi_0^0 + \pi_0^0. \tag{9b}$$

Indeed, any reaction is possible only if the sum of the "mass numbers" on either side is either even or odd. It is consistent with the nature of the electromagnetic

couplings to assign to the photon a mass number zero. The mass number rule thus also excludes decays of the types (7) to (9) with an arbitrary number of photons added on the right-hand side of the reaction. Furthermore, if

$$\pi_1^0 \rightarrow \pi_0^0 + \pi_0^0 + \pi_0^0 \tag{10a}$$

$$\pi_1^0 \rightarrow \pi_0^+ + \pi_0^- + \pi_0^0 \tag{10b}$$

were energetically and otherwise possible, it would still be a forbidden transition.

In concluding this section we note that so far no dissymmetry has explicitly been brought in between charged and neutral V's. We have seen that nevertheless it is conceivable that fewer N_1^{ch} may be produced than N_1^0 if only the N_1^- is treated on the same footing as the proton. But even apart from this possibility the production rate of V_0 and V_{ch} need not necessarily be equal. For example, a symmetrical theory of the π_0-nucleon coupling gives at least near threshold, dissimilarities in the π_0^0 and $\pi_0^\pm$-production, a.o., because of the exclusion principle, and similar effects may occur for the V's. In the subsequent sections we will, however, assume the productions to be equal and investigate whether the decay processes alone can account for the dissymmetry. It will turn out that such a dissymmetry may exist for $\pi_1^\pm$- *vs* π_1^0-decay, but not for $N_1^\pm$- *vs* N_1^0-decay.

III. THE DECAY MECHANISMS OF THE *V*-PARTICLES

To get sufficiently slow decay we now have to assume that the $(N_i N_j \pi_k)$ are extremely weak (and thus insignificant for the production) if

$$i + j + k = \text{odd}. \tag{11}$$

Condition (11) singles out the interactions

$$(N_1 N_0 \pi_0), \tag{12a}$$

$$(N_0 N_0 \pi_1), \tag{12b}$$

$$(N_1 N_1 \pi_1), \tag{12c}$$

and we will again introduce one coupling constant, g, to be representative of the whole group. As before, g indicates a general order of magnitude. The couplings (12) make possible even-odd transitions in the mass number.

The decays are generally brought about by combinations of the interactions (2) and (12). Most of the processes to be considered give divergent results for the corresponding matrix element. But even apart from that, a determination of the magnitude of g is hampered by the simultaneous dependence of the decay probabilities on the large coupling constant G. In this respect the situation is similar to that for neutrino processes where a coupling constant occurs which is very small, but the precise value of which likewise depends on G. Here we shall essentially always need the g^2-dependence to make the decay rate slow. In what follows we will

confine the discussion to the lowest order matrix elements and will indicate the kinds of questions one runs into in analyzing the decay problems. With due reservations one estimates in this way

$$g^2/4\pi\hbar c \sim 10^{-11}. \tag{13}$$

(A) The Decay of the V_0's

(a) N_1^0-Decay

This is possible through (12a). This coupling comprises two possibilities:

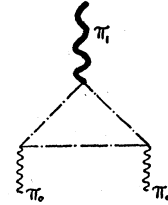$$N_1^0 \rightarrow N_0^+ + \pi_0^-, \tag{14a}$$

$$N_1^0 \rightarrow N_0^0 + \pi_0^0 (\rightarrow N_0^0 + 2\gamma). \tag{14b}$$

The π_0 are pseudoscalar mesons and we shall take the coupling (12a) to be of γ_5-type. The lifetime for either process (14) is then given by

$$\tau^{-1} \approx \frac{g^2}{4\pi\hbar c} \cdot \left(\frac{2\mu_0}{M_0}\right)^{\frac{1}{2}} \left(\frac{M_1 - M_0 - \mu_0}{M_0 + \mu_0}\right)^{\frac{1}{2}} \cdot \frac{\mu_0 c^2}{\hbar} \text{ sec}^{-1}. \tag{15}$$

Here p, E are the momentum and energy of the nucleon; M_1, M_0, μ_0 are the masses of N_1, N_0, and π_0, respec-

Fig. 2. Graph for the $2\pi_0$-decay of the π_1.

tively. With $M_1 = 2200m$, $M_0 = 1836m$, $\mu_0 = 276m$:

$$\tau^{-1} \sim (g^2/4\pi\hbar c) \times 10^{21} \text{ sec}^{-1}, \quad \text{so} \quad g^2/4\pi\hbar c \sim 10^{-11}$$
$$\text{for} \quad \tau \sim 10^{-10} \text{ sec.}$$

It may be remarked that the decay (14b) is invisible in a cloud chamber. It is of course true that the γ-rays resulting from the π_0^0 decay are shower producing, but it will in general not be possible to relate such showers to (14b) as the origin of the initial photons. Assuming the g's responsible for (14a) and (14b) to be of the same order of magnitude, this means that an estimate of the production probability of the N_1^0 on the basis of the frequency of the event (14a) would be underestimated by a factor ~ 2.

Radiative decays are also $\sim g^2$ and further $\sim e^2$. They are, therefore, insignificant.

(b) The π_1^0-Decay

In discussing the decay modes of the π_1^0 it now becomes indispensable to specify in further detail the nature of the π_1^0. We will first of all investigate the possibility that the π_1^0 decays into two π_0's. Now as the latter are pseudoscalar this makes it certainly necessary to assume the π_1^0 to be nonpseudoscalar and

nonpseudovector. In the present section this assumption will be made. But in Sec. V an investigation will be made of what the consequences would be if $\pi_1{}^0$ were pseudoscalar all the same.

Let us now, therefore, admit the $\pi_1{}^0$ to be scalar or vector and analyze the decays

$$\pi_1{}^0 \rightarrow \pi_0{}^+ + \pi_0{}^-, \tag{16a}$$

$$\pi_1{}^0 \rightarrow \pi_0{}^0 + \pi_0{}^0(\rightarrow 4\gamma). \tag{16b}$$

The simplest graph which describes the processes (16) is indicated in Fig. 2. The dot-dashed lines mean

TABLE I. Contribution of triangle graph to process (16a).

T_1	SS	SV	VV	VT
1	y	n	n	n
τ_3	n	n	y	y

that we have either an N_0 or an N_1-line. This depends on what particular combinations of interactions to be chosen from among (2) and (12) are operative in the vertices 1, 2, 3, respectively. The reader will easily verify that for any combination so chosen, the matrix element is always proportional to either gG^2 or g^3. This is, in fact, a consequence of the even-odd rules for the interactions and of the circumstance that on the left (right) of (16) we have total mass number 1(0). Only the gG^2-transitions are relevant, of course. And to any order the decay matrix element is proportional to at least the first power of g.

Provided the transition is not forbidden by any selection rule not yet mentioned, it follows from the work of Fukuda, Hayakawa, and Miyamoto[23] and of Ozaki, Ôneda, and Sasaki[24] that the lifetime is of the order of

$$\tau^{-1} \sim (10^{19} \text{ to } 10^{21}) \cdot (G^2/4\pi\hbar c)^2 \cdot (g^2/4\pi\hbar c). \tag{17}$$

The authors mentioned study decays of a Boson (with a mass comparable to that of $\pi_1{}^0$) into $\pi_0{}^+ + \pi_0{}^0$ (in the present notation), for all combinations of couplings of the (Boson, $\pi_0{}^+$, $\pi_0{}^-$)-system. With few exceptions the matrix elements are divergent, and regulator techniques were used to obtain a covariant cutoff.

TABLE II. Contribution of triangle graph to process (20).

T_0	SS	SV	VV	VT
1	y	n	n	n
τ_3	n	n	y	y

Quite apart from the question of trustworthiness of this procedure, the general orders of magnitude found seem to be reasonable when comparing the answers with the (finite) outcome from the $\pi_0{}^0 \rightarrow 2\gamma$ decay by means of simple dimensional arguments. For the decays (16) one obtains similar results.

In order to estimate g from (17) and the experimental τ, a knowledge of G is required. From the $(N_0 N_0 \pi_0)$ interaction it seems indicated that $G^2/4\pi\hbar c \gtrsim 1$, but not very much more is known owing to the aforementioned inadequacy of our methods in dealing with strongly coupled systems. However this may be,

orders of magnitude of g as given by (13) seem again to be involved.

It must be added that there are further rules which in certain instances can inhibit the reactions (16a; b). Considering first (16a), if in the vertices 1, 2, 3 couplings are operative such that

$$n(v) + n(t) + n(\tau_3) = \text{odd}, \tag{18a}$$

then the contribution of the corresponding graph vanishes. Here $n(v)$ and $n(t)$ denote the total number of vector and tensor couplings, respectively, occurring in the triangle; $n(\tau_3)$ is the total number of neutral Boson couplings involving τ_3. The rule (18a) is a generalization of the Furry theorem and was first noted by Fukuda and Miyamoto.[25] It will be referred to as the first FM theorem. With the help of (18a) Table I has been constructed, which collects the allowed and forbidden transitions (16a) as calculated from the triangle graph. Notation: y=allowed, n=forbidden; SS=scalar $\pi_1{}^0$ with scalar coupling, etc.; T_1 indicates the isotopic spin character of $\pi_1{}^0$. The table holds for any choice of combinations of N_1, N_0 lines in the triangle, which need not all have the same couplings, of course; but it seems not yet worth while to make any further classification in that direction. Note that the innocuous SV case must have n throughout.[26]

As pointed out in reference 25, (18a) holds for any closed Fermion polygon in any graph, provided that the Bosons emerging from the vertices are not all neutral. In the latter case, of which (16b) is an instance, there is another rule which applies and which is[27]

$$n(\tau_3) = \text{odd is forbidden};$$
$$\text{if } n(\tau_3) = \text{even}, \quad n(v) + n(t) = \text{odd is forbidden}, \tag{18b}$$

(second FM theorem). Hence, the process (16b) is only allowed for a scalar $\pi_1{}^0$ with scalar coupling, irrespective of the isotopic spin characters.

The preceding brief remarks may exemplify the role of the FM theorems in analyzing the contributions of a particular kind of graph to some given process. However, the circumstance that the simple graph of Fig. 2 gives no contribution to the matrix element of the corresponding process does not necessarily mean that an absolute selection rule obtains. By using electromagnetic interactions in intermediary states it can, e.g., be shown that the process (16a) is not rigorously forbidden even if $\pi_1{}^0$ is a vector meson with $T_1 = 1$. A general discussion of the connection between the Furry- and the FM-theorems on the one hand and the existence of absolute selection rules on the other will be given elsewhere.

The $2\pi_0$ decay rate is obviously favored compared to the (π_0, γ) and the 2γ processes in the ratio $G^4 : : G^2 \epsilon^2 : : \epsilon^4$, independent of g. While this argument in itself has enough strength, it may yet be of interest to remember that there are additional selection rules: If $\pi_1{}^0$ is a vector-Boson one has the well-known forbiddenness of 2γ-decay; if it is a scalar then $(\pi_0{}^0, \gamma)$ is forbidden for covariance reasons.

[23] Fukuda, Hayakawa, and Miyamoto, Prog. Theor. Phys. 5, 283, 352 (1950).
[24] Ozaki, Ôneda, and Sasaki, Prog. Theor. Phys. 4, 524 (1949); 5, 25, 165 (1950).
[25] H. Fukuda and Y. Miyamoto, Prog. Theoret. Phys. 4, 389 (1950). The relation of the theorems mentioned here and a generalized concept of charge conjugation is discussed by K. Nishijama, Prog. Theor. Phys. 6, 614 (1951). These theorems are valid whether or not the Fermions in the closed loop have all the same mass.
[26] See F. J. Dyson, Phys. Rev. 73, 929 (1948).
[27] In reference 25 it is incorrectly stated that the rule is $n(v) + n(t) = \text{odd}$ is forbidden if only neutral mesons are involved. The relations (18b) follow immediately, however, from the argument there presented. A third general rule could still be added: If only neutral mesons and photons are involved we have

$$n(v) + n(t) = \text{odd is forbidden.} \tag{18c}$$

The latter rule is actually the one closest to the familiar Furry theorem. See W. H. Furry, Phys. Rev. 51, 125 (1936).

Thus, we have obtained a scheme in which there are two slow two-body decay rates, one for N_1^0 and one for π_1^0. The decay products are nucleons and π-mesons. In Sec. IV we will investigate other V_0-decay mechanisms, but we will now first discuss the $V_{\rm ch}$-phenomena.

(B). The $V_{\rm ch}$-Decay

The analogs of (14) and (16) for N_1^+, π_1^+ are

$$N_1^+ \rightarrow N_0^0 + \pi_0^+, \qquad (19a)$$

$$N_1^+ \rightarrow N_0^+ + \pi_0^0, \qquad (19b)$$

$$\pi_1^\pm \rightarrow \pi_0^\pm + \pi_0^0. \qquad (20)$$

Now if it is true that all G's on the one hand and all g's on the other are of the same order of magnitude, then (19) and (20) give τ's of the same order of magnitude, if not equal, as for (14) and (16). This leads to a contradiction with the present experimental evidence: one would observe more than $N_1^{\rm ch}$- than N_1^0-decays, (14b) being invisible. There are no selection rules for these processes. With regard to (20), the allowed and forbidden transitions are listed in Table II. T_0 is the isotopic spin character of π_0^0. Comparison with Table I shows some possibilities of a dissymmetry between V_0 and $V_{\rm ch}$ decays. But it throws no light on the observed preponderance of the former, which can in fact only be understood by assuming either less production or the existance of decay processes for the $V_{\rm ch}$ faster than those considered so far. In the next section it will be shown that the latter is not inconceivable.

IV. PROCESSES INVOLVING μ-MESONS

So far no decays have been considered involving μ-mesons. Now that we have discussed a model in which it is feasible, at least for V_0, to have a reasonable rate of production vs decay without them (and this seemed at first sight to be the greatest stumbling block), we have to ask how decays involving μ's compete with those considered so far. Such a competition has to be anticipated, as lifetimes $\sim 10^{-10}$ sec are just what one would expect for processes involving neutrinos.[9]

We first investigate π_1 decays involving neutrinos and do this in analogy with the corresponding $\pi_0^\pm$-process. It is well known that the situation here is not altogether satisfactory. In particular, it has so far not been possible to explain the $\pi_0^\pm$ decay without invoking Fermi couplings which give divergent results, on any coupling scheme, for the $\pi_0^\pm$-$\mu^\pm$ decay. On the other hand, it has been shown[28] that the $(\pi_0\mu)$ vs $(\pi_0 e)$ rate can be understood to be rather large ($\sim 10^4$) and this result, owing to a particular choice of interactions (PS-PS or PS-PV π_0 and PV-Fermi coupling), can be stated independently of the divergent integrals involved and is so far not in contradiction with experiment.[29] Anyhow, not having better suggestions, we shall follow the analogous procedure for π_1, i.e., use the $(N_i N_j \pi_k)$ couplings in conjunction with Fermi couplings between $(N_i N_j)$ and the light Dirac particles.

The type of graph, then, which is involved in the process

$$\pi_1^\pm \rightarrow \mu^\pm + \nu \qquad (21)$$

is drawn in Fig. 3, the dashed lines indicating the (μ, ν) pair. In vertex 2 the weak Fermi interaction operates. Hence, in order to get relevant decay rates we can use in vertex 1 only G- (not g-) interactions. This has an immediate consequence which we shall discuss further below. Choosing in 1 the interaction $(N_1 N_0 \pi_1)$ which is the only G-coupling involving π_1, it follows that the N_1 are unstable themselves in virtue of the same interaction in 2 which we need for the process (21).

Before continuing the discussion of (21) we remark that analogous processes can be conceived for the π_1^0, the emerging Fermions being neutral or oppositely charged. But if the isotopic spin character of the interactions in 1 and 2 is opposite, then the graph does not contribute according to the second FM theorem applied to the closed loop. Other sets of intermediate states through which the process might occur are seen to be irrelevant as they give too slow decays. Thus, choosing the isotopic spin dependences appropriately allows us to disregard this type of π_1^0-decay. This is a possible means to obtain dissymmetry between V_0- and $V_{\rm ch}$-decay.

In estimating the decay rate of (21) we will attempt to free ourselves, as well as is possible, from the occurrence of the multiplicative divergent factor due to the integration over the momenta of the closed loop. This cannot be done in exactly the same way as for $(\pi_0\mu)$ vs $(\pi_0 e)$ decay where this factor drops out in the ratio as long as one takes the same couplings for both processes; we know the π_0^+ to be pseudoscalar whereas it has been assumed throughout that the π_1^+ is nonpseudoscalar. However, there is still one (and only one) other possibility:[30] If the π_1^+ is a VV-meson and the interaction in 2 a tensor-coupling, we get the same divergent integral[31] as for the PS-PS π_0^+ with PV-Fermi coupling.

FIG. 3. Graph for the
μ-decay of the π_1.

In this way we find for the ratio of $(\pi_1^+$-$\mu)$ decay vs $(\pi_0^+\mu)$ decay, for equal strength of the Fermi interactions (μ is the μ-meson mass):

$$\frac{\tau^{-1}(\pi_1^+\mu)}{\tau^{-1}(\pi_0^+\mu)} = \frac{4}{3} \cdot \frac{\mu_0^3}{\mu_1^3 \mu^2} \cdot \frac{(\mu_1^2 - \mu^2)^2}{(\mu_0^2 - \mu^2)^2} \cdot (\mu_1^2 + 2\mu^2) \approx 300.$$

The lifetime of π_0^+ being[32] $1.8 \cdot 10^{-8}$ sec, we get a value of $\sim 10^{-10}$ sec for (21). It need hardly be added that the present argument does not pretend to show the preference of one $\pi_1^\pm$-coupling over another or to give a theoretically reliable value for the $(\pi_1\mu)$ decay lifetime; it merely illustrates the point of competition between $(\pi_1\mu)$ and $(\pi_1, 2\pi_0)$ decay. Nor does it seem profitable at this stage to discuss in any detail the occurrence of π_1-electron decay which, with the coupling used above, would be only slightly slower than (21); this process will not be taken into consideration hereafter.

It should now be observed that if $\tau(\pi_1^+, \mu)$ $\lesssim \tau(\pi_1^+, 2\pi_0)$, the effective life time of π_1^+ decay is smaller than that for π_1^0. In fact, if the inequality holds strongly one will observe

$$\pi_1^\pm \rightarrow \mu^\pm + \nu, \quad \pi_1^0 \rightarrow \pi_0^+ + \pi_0^-,$$

as principal π_1-processes, the former being a relatively rare event compared to the latter.

[28] M. Rudermann and R. Finkelstein, Phys. Rev. 76, 1458 (1949); S. Nakamura et al., Prog. Theor. Phys. 5, 740 (1950).

[29] H. L. Friedman and J. Rainwater, Phys. Rev. 84, 684 (1951).

[30] See J. Steinberger, Phys. Rev. 76, 1180 (1949).

[31] This is not strictly correct. The divergent integral depends itself on the mass of the decaying particle. However, this is presumably a slow dependence if an effective cutoff would exist which, in mass units, is large compared to the π_k-mass.

[32] Jakobson, Schulz, and Steinberger, Phys. Rev. 81, 894 (1951).

As already pointed out we are now also forced to consider the direct processes

$$N_1{}^0 \rightarrow N_0{}^+ + \mu^- + \nu, \tag{21a}$$

$$N_1{}^\pm \rightarrow N_0{}^0 + \mu^\pm + \nu, \tag{21b}$$

(two neutrino- and electron-neutrino processes are again ignored). These decays have a characteristic time given by

$$\tau^{-1} \sim (g_F/\hbar c)^2 \cdot (\mu c/\hbar)^4 \cdot (\mu c^2/\hbar) \cdot \sec^{-1},$$

where g_F is the Fermi constant involved. Giving this quantity the same value as for processes involving N_0's only, namely[33] $\approx 2 \times 10^{-49}$ erg cm³, one has $\tau \approx 0.6 \times 10^{-10}$ sec. Hence, competition between these decays and the two-body decays must be expected. However, it does not seem possible to have any dissymmetry between (21a) and (21b).

Summarizing the results obtained with the present model, we have:

(a) The V_0 decays consist of three processes, one for the particle with mass $\sim 800m$:

$$\pi_1{}^0 \rightarrow \pi_0{}^+ + \pi_0{}^-;$$

two for the particle with mass $\sim 2200m$:

$$N_1{}^0 \rightarrow N_0{}^+ + \pi_0{}^-, \quad N_1{}^0 \rightarrow N_0{}^+ + \mu^- + \nu.$$

The latter might account for noncoplanar events. An estimate of the branching ratio clearly involves too much theoretical arbitrariness.

(b) If the burden of explaining the dissymmetry of V_0- with $V_{\rm ch}$-decay rests with the decay mechanisms rather than with the production process, the model can account for a dissymmetry between $\pi_1{}^0$-decay and

$$\pi_1{}^\pm \rightarrow \mu^\pm + \nu.$$

It cannot[34] in any natural manner describe a dissymmetry in the behavior of $N_1{}^0$ and $N_1{}^\pm$ insofar as their disintegration is concerned, but it is possible that the $N_1{}^{\rm ch}$ are produced less copiously than the $N_1{}^0$ without the explicit introduction of disparities in the corresponding G's (see Sec. II). (*Added in proof*: decays like $\pi_1{}^0 \rightarrow \pi_0{}^\pm + \mu^\mp + \nu$ and $\pi_1{}^\pm \rightarrow \pi_0{}^0 + \mu^\pm + \nu$ can be discussed by similar methods.)

V. PSEUDOSCALAR HEAVY BOSONS

The assertion that $\pi_1{}^0$ is nonpseudoscalar is based on the presumed two-body decay of this particle into two π-mesons. The present experimental situation does not make it definitely superfluous to explore what would happen if the $\pi_1{}^0$ were pseudoscalar after all and the $2\pi_0$ decay therefore forbidden. Quite apart from this we know from the detailed discussion of the τ-meson decay, given by Powell,[35] that this process

very probably is

$$\tau^+ \rightarrow \pi_0{}^+ + \pi_0{}^+ + \pi_0{}^-, \tag{22}$$

and this in turn means that the $\tau^\pm$ may be pseudoscalar; (it may also be pseudovector). We will discuss below whether or not a pseudoscalar π_1 might conceivably be related to the τ-meson. Before doing this we shall first discuss, in a general manner, the decay schemes of a heavy neutral or charged pseudoscalar Boson which we provisionally denote by B^0, $B^\pm$, respectively. The mass of these particles is taken to be $< 4\mu_0$.

B^0 can of course decay according to

$$B^0 \rightarrow 2\gamma. \tag{23}$$

Apart from $B \rightarrow 2\pi_0$, also

$$B^\pm \rightarrow \pi^\pm + \gamma$$

is forbidden on gauge invariance grounds.[23,24] Hence, we are left with the following processes for B^0:

$$B^0 \rightarrow \pi_0{}^+ + \pi_0{}^- + \gamma, \tag{24}$$

$$B^0 \rightarrow \pi_0{}^0 + \pi_0{}^0 + \gamma, \tag{25}$$

$$B^0 \rightarrow \pi_0{}^+ + \pi_0{}^- + \pi_0{}^0, \tag{26}$$

$$B^0 \rightarrow \pi_0{}^0 + \pi_0{}^0 + \pi_0{}^0. \tag{27}$$

(25) is readily seen to be ruled out by the Furry theorem.[36] As to the possibility that the FM theorems can impose restrictions, the situation is this:

Process (24): In the vertex of the Fermion square in which the photon emission operates, there occurs a factor $\frac{1}{2}(1 + \tau_3)\gamma_\mu$ which has, from the point of view of relation (18a), an even as well as an odd part. Hence, whatever happens in the other vertices, one can never get a selection rule from the first FM theorem.

Processes (26), (27): These are allowed if B^0 and π^0 have the same, forbidden[37] if they have different isotopic spin characters 1 or τ_3. (27) is of course "invisible."

Provided (26) and (27) are allowed, these decays are favored compared with (24) in the ratio $G^2 :: e^2$. On the other hand, the phase space V_γ for reaction (24) is larger than that (V) for (26), (27). Treating the π_0's in the latter nonrelativistically, one has

$$\frac{V_\gamma}{V} = \frac{\sqrt{3}}{2\pi} \cdot \left(\frac{\mu_1}{\mu_1 - 3\mu_0}\right)^2 \cdot Z,$$

$$Z = \int_0^P p^2 dp (\mu_1 - 2E)^2 \left[\frac{3\{(\mu_1 - E)^2 + E^2\}}{\{(\mu_1 - E)^2 - p^2\}^2} - \frac{4p^2\{(\mu_1 - E)^2 - E^2\}}{\{(\mu_1 - E)^2 - p^2\}^3} \right],$$

$$E = (p^2 + 1)^{\frac{1}{2}}, \quad P = \left(\frac{\mu_1{}^2}{4\mu_0{}^2} - 1\right)^{\frac{1}{2}}.$$

[33] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 144 (1949).

[34] In this work further differentiations which may occur as a result of the Yang-Tiomno classification of the reflection properties of spinors have not been considered. See C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950).

[35] See Powell *et al.*, Phil. Mag. **42**, 1040 (1951).

[36] See relation (18c) of footnote 27.

[37] That is to say, the graph with a Fermion square gives no contribution. Process (26) can nevertheless take place by letting an intermediary photon create the ($\pi_0{}^+$, $\pi_0{}^-$)-pair; but the probability for this process is small compared to that of (24), which is all that is needed for the argument developed hereafter.

Here μ_1 is the B^0-mass. Taking the latter to be of the order of the τ-meson mass ($\approx 960m$), $V_\gamma/V \approx 7$ which does not offset the G^2/e^2 balance in favor of (26), (27).

For the charged B's the reactions are

$$B^+ \rightarrow \pi_0^+ + \pi_0^{0-} + \gamma, \tag{28}$$

$$B^+ \rightarrow \pi_0^+ + \pi_0^0 + \pi_0^0, \tag{29}$$

$$B^+ \rightarrow \pi_0^+ + \pi_0^+ + \pi_0^-. \tag{30}$$

There are no selection rules. Reaction (28) is slow compared with (29) and (30). Reactions (26), (27), (29), and (30) can be expected to have comparable lifetimes provided the former two are not forbidden. The lifetime for reaction (30) was first calculated by Sheila Power[38] and is extensively discussed in references 23 and 24.

After these preliminaries, let us now inquire whether an identification of B and π_1 is possible. Clearly the crucial point is whether (26) and (27) are allowed or not. In this connection it should first of all be noted that if a mass $\sim 800m$ is estimated on an assumed two-body decay which actually involves an invisible particle as well, this value is bound to be too low. We should thus distinguish two cases:

(a) $\mu_1 < 3\mu_0$. Reactions (26) and (27) are energetically forbidden. The π_1^0 decay is given by (24), which process is discussed in more detail below. Assuming, as we have done throughout, the mass of $\pi_1^\pm$ not to differ much from that of π_1^0, (29) and (30) are forbidden too. It would not be possible to identify $\pi_1^\pm$ with the τ-meson.

(b) $4\mu_0 > \mu_1 > 3\mu_0$. This case has further to be subdivided as follows:

(α) Reactions (26) and (27) are allowed. The radiative decays can be ignored. The identification of $\pi_1^\pm$ with the τ-meson is possible. All π_1 decays become phenomena as rare as the τ-meson decay (assuming again no disparities in the production). They would manifest themselves thus:

Process (26): a noncoplanar decay into charged π-mesons, a pair of γ-rays from the π_0^0 decay accompanying the event.

Process (29): of the V_{ch} decay type, noncoplanar, four γ-rays accompanying the event.

Process (30): the τ-meson decay.

The main observed V_0 decays would now be those of N_1^0, involving either π_1^0 or μ.

(β) Reactions (26) and (27) are forbidden. Now (24) becomes important. There is now also a marked dissymmetry between the π_1^0, the decay of which is accompanied by a photon, while this is not so for $\pi_1^\pm$. The latter is now faster, as we have seen, than the π_1^0 decay by a factor $\sim(1/7)G^2/e^2$.

It is thus not without interest to calculate in detail the probability of process (24). The graphs for it are

given[39] in Fig. 4. The relative weight of graphs (II) and (I) is as 2:1. All Boson couplings have been taken as PS-PS. The contributions (apart from weight) of graphs (I) and (II) turn out to be finite and equal in absolute magnitude. They have opposite (equal) sign if the isotopic spin character of π_1^0 is $1(\tau_3)$. We can thus, e.g., assign the character 1 to π_1^0, and τ_3 to π_0^0, in order to forbid (26) and (27).

One further verifies that there are various possibilities of assigning N_1 or N_0 to the Fermion lines. Apart from terms $\sim(M_1 - M_0/M_0)$ these give the same contribution. We take one representative sample of Fermion lines. The effective energy density for the process on hand turns out to be ($\hbar = c = 1$)

$$-\frac{gG^2\epsilon}{3\pi^2 M^3}\phi_1 \frac{\partial\phi_0}{\partial x_\lambda} \frac{\partial\phi_0^*}{\partial x_\tau} \frac{\partial\phi_\sigma}{\partial x_\rho}\epsilon^{\lambda\rho\sigma\tau}. \tag{31}$$

Here ϕ_1 is the π_1-wave function, ϕ_0 that of the (charged) π_0, and ϕ_σ the electromagnetic potential. $\epsilon^{\lambda\rho\sigma\tau} = +1(-1)$ for even (odd) permutations of (1, 2, 3, 4). (31) satisfies all invariance requirements.
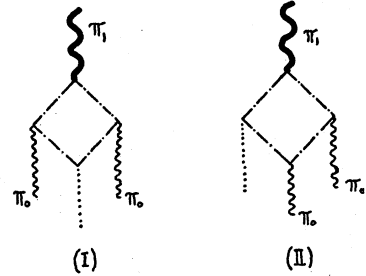

FIG. 4. Graphs for the $2\pi_0 + \gamma$-decay of the π_1.

(I)   (II)

We obtain for the absolute square of the matrix element H of the process, after summing over the polarizations of the photon,

$$|H|^2 = \frac{g^2 G^4 e^2 \mu_1}{72\pi^4 M^6} \cdot \frac{p_+^2 p_-^2 \sin^2\theta}{E_+ E_- k}.$$

Here $p_\pm$ is the momentum of the $\pi_0^\pm$, θ is the angle between them and k is the energy of the produced photon. The probability of the decay is found to be

$$\tau^{-1} = \frac{8}{27\pi^3} \cdot \frac{g^2}{4\pi} \cdot \left(\frac{G^2}{4\pi}\right)^2 \cdot \frac{e^2}{4\pi} \cdot \left(\frac{\mu_1}{M}\right)^4 \cdot \left(\frac{\mu_0}{M}\right)^2 \cdot \mu_0 \cdot f \text{ sec}^{-1},$$

where

$$f = \int_0^P F(p)dp, \quad P = \left(\frac{\bar{\mu}_1^2}{4} - 1\right)^{\frac{1}{2}}, \quad \bar{\mu}_1 = \frac{\mu_1}{\mu_0}.$$

[38] S. Power, Phys. Rev. 76, 865 (1949).

[39] It is readily seen that graphs obtained from Fig. 2 by attaching an arbitrary number of photon lines to the π_0-lines all give zero.

Here

$$F(p) = p^4 \{\bar{\mu}_1{}^2 - 2\bar{\mu}_1(p^2+1)^{\frac{1}{2}}\}^3 (p^2+1)^{-\frac{1}{2}}$$
$$\cdot \{\bar{\mu}_1{}^2 + 1 - 2\bar{\mu}_1(p^2+1)^{\frac{1}{2}}\}^{-2}$$

is the distribution of the momenta (in units $\mu_0 c$) of one of the emerging charged particles. The distribution is much more peaked and the maximum lies at higher momenta than one would expect from the statistical factor only. It is somewhat interesting to note that if the $\pi_1{}^0$ mass is taken $\approx 960m$, the optimum of p lies quite near the value for the momentum of the π_0 in the process (16a), provided there the mass of $\pi_1{}^0$ is $\sim 800m$. For $\mu_1 \approx 960m$ one finds for the rate of decay of (24):

$$\tau^{-1} \sim \frac{g^2}{4\pi} \cdot \left(\frac{G^2}{4\pi}\right)^2 \cdot 10^{17} \text{ sec}^{-1}.$$

As was to be expected, one now gets an estimate for $g^2/4\pi$ which is much larger than the one given by (13) but still many orders of magnitude smaller than the fine structure constant, so that the qualitative features of the foregoing discussion remain essentially unaltered.

While it may be premature to decide whether the π_1 is pseudoscalar or not, it may be useful to stress these two main features of model $(b\beta)$:

(a) The $\pi_1{}^0$ decay should be accompanied by a γ-ray. For given momentum p of, say, $\pi_0{}^+$, the energy of the photon lies between K_+ and K_-, where

$$K_\pm = 1/2 \cdot \mu_1(\mu_1 - 2E)(\mu_1 - E \pm p)^{-1}, \quad E = (p^2 + \mu_0{}^2)^{\frac{1}{2}}.$$

At the optimal value of p, these limits are ~ 50 and 150 Mev, respectively.

(b) If the $\pi_1{}^+$ is identified with the τ-meson, the lifetime for the latter should be shorter than that of the $\pi_1{}^0$.

It may finally be pointed out that if this identification is not possible, one may still expect V_{ch}-decays of the type (29) and, if the neutral counterpart of the τ-meson exists, rare V_0-decays of the type (26).

VI. CONCLUDING REMARKS

Whereas in the present model only interactions of a familiar form are employed, coupling constants g be-tween heavy Fermions and Bosons occur with a novel magnitude. Such very weak couplings have so far only been considered in neutrino processes, such as β-, μ-, π-decay, etc. A consistent description of these disintegrations themselves in terms of $\bar{\psi}\psi\phi$-interactions (which, though it may be prejudice, one likes to consider as the primary structure generally) has so far not been given. The existence of the new particles and of new interactions between them may perhaps shed light on this question.

Even if one accepts the premises from which the present scheme starts, it can lay no claim to uniqueness. A main reason for this is that other particles, so far unobserved because of shorter lifetimes, for example, might very well enter the picture. As pointed out in Sec. II, one will then have to consider whether such regularities as have here been noted can still be upheld or should be replaced by others. The search for ordering principles at this moment may indeed ultimately have to be likened to a chemist's attempt to build up the periodic system if he were given only a dozen odd elements. The author would, therefore, like to stress that this work should be considered as representing a general point of view according to which one may attempt to codify the present information, rather than as an unflexible proposal for a particular set of interactions.

In the model here discussed, the heavy V cannot be considered as an isobar in the strong coupling sense; yet it seems closely related to the nucleon. A somewhat similar situation is met in comparing the μ-meson with the electron. Also these particles have markedly related properties, while their weak couplings, their large mass difference, and the long lifetime of the μ-meson render an isobar picture implausible. This is perhaps an indication of the existence of families of elementary particles (like a nucleon and an electron family) in which, not unlike the levels in a given kind of atom, the members of a given family are distinguished from each other through a quantization process, but one of a new kind.